EGH 445: Modern Control:

State- Feedback Control of Cart-Pendulum System

# Abstract

This report attempts to provide a technical description of the development and results of the state-feedback control design for the cart-pendulum system developed in the computer for only the stabilisation of the pendulum in the upright position.

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# Introduction

The purpose of this experiment is to model the active control of the cart-pendulum system about the two equilibrium points, where the pendulum angle is at 0 radians (where the pendulum is vertically upright) or at π radians (where the pendulum is hanging downwards) from different starting cart positions and pendulum angles.

# Building the State-Feedback Control of the Cart-Pendulum System:

## Building a Model for the Cart-Pendulum:

The idealised model of the cart-pendulum system can be idealised as a pendulum of mass *m* and length l attached to a cart of mass Mc. The pendulum moves under the action of gravity and the cart moves only on the horizontal direction and is actuated by the control force F. The state-space model can be written as follows:

Where the states are:

* the position of the cart
* the angle of the pendulum
* the velocity of the cart
* the angular velocity of the pendulum

With the values of the model parameters given in Table 1.

|  |  |
| --- | --- |
| Parameter | Value |
|  |  |
|  |  |
|  |  |
|  |  |

### Linearised System:

### Non-Linearised System:

## Building a Model for the Control System:

The cart-pendulum system was used as a benchmark to design controllers based on state feedback. We can consider the stabilisation problem of two desired equilibriums of the cart-pendulum system. The equilibriums are :

; , however this report will only consider the stabilisation of the pendulum in the upright position,

In order to find the state-feedback controllers in the form

To stabilise the equilibrium where the input u is the control force,

### Linearised System:

### Non-Linearised System:

## Building a Model for the Observer System:

The objective is to build observers in the general form:

Where is the state estimation, is the output estimation, and are the input and output of the system respectively. The matrix is the observer gain that has to be selected to ensure convergency of the observer error to zero. This is achieved in MATLAB® by using the place command as follows: , . This command finds the closed-loop pole assignment using state-feedback by computing a state-feedback matrix such that the eigenvalues of A-B\*K are those specified in the vector . No eigenvalue should have a multiplicity greater than the number of inputs.

### Linearised System:

### Non-Linearised System:

# Conclusion:

The cart-pendulum system was first modelled based on idealised assumptions and controlled with a theoretical controller of zero time-response and observed with an observer that can accurately measure the position of the cart and the angular displacement of the pendulum. While the simulations of the cart-pendulum in MATLAB® seem to behave as expected, this idealised version does not account for real-world effects such as the pendulum not being attached with a massless rod and that the cart is not restricted in the vertical plane, which may lead the real-world behaviour of the cart-pendulum system behaving wildly different from the simulated model and thus the controller and observer gains might need to be re-calculated to account for these discrepancies.